Multiplicative Reasoning – the Key Concepts

Aims:

- To understand the importance of repeated addition, scaling, grouping and sharing and its links multiplicative reasoning.

- To understand how to support children in your setting to develop a deeper understanding of multiplicative reasoning in KS2 by exposing the structure of the maths.

Aims of the National Curriculum

Fluency

Reasoning

Maths

Problem solving

Procedural Fluency

Conceptual Understanding
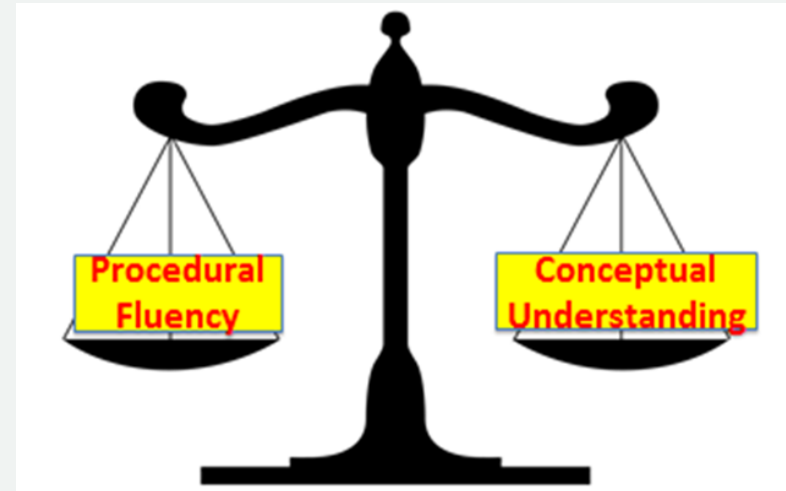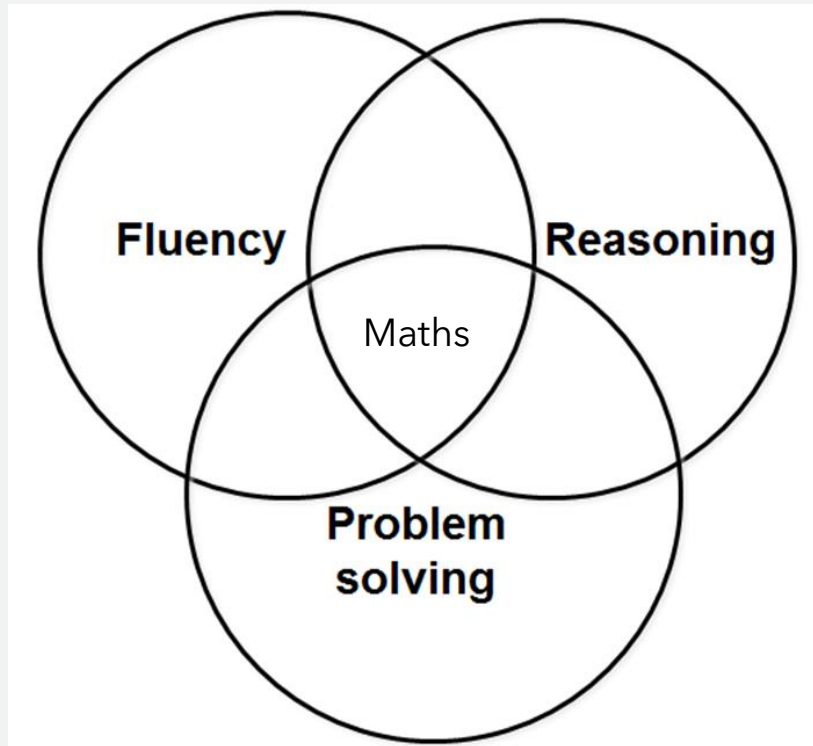
Connections model

(Haylock and Cockburn; 2008)
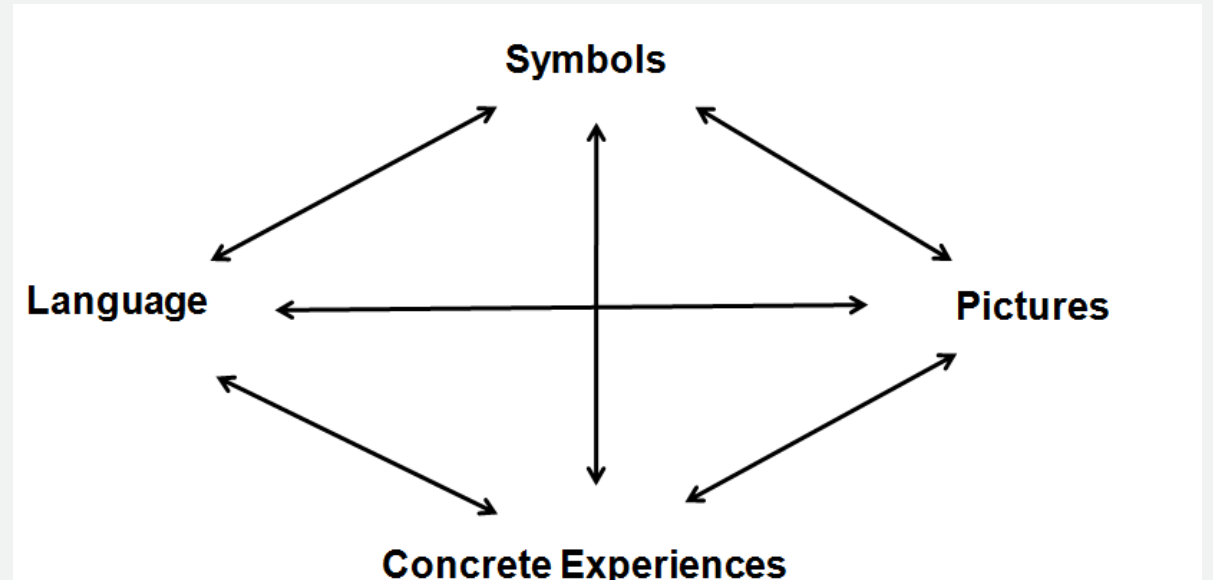
What do you think?

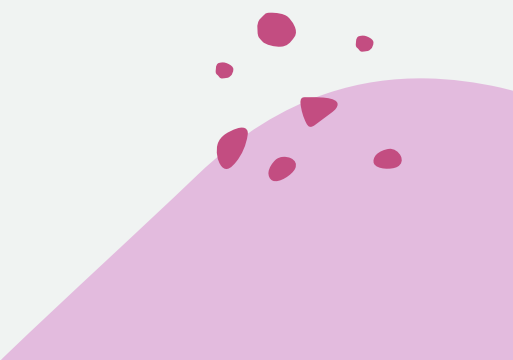# Young children are differently experienced and not differently able at mathematics

Gripton, C and Williams, H; The principles for appropriate pedagogy in early mathematics: Exploration, apprenticeship and sense-making. Part 1; ATM Journal 285; Feb. 2023

Multiplicative reasoning, what does this mean?

How is this the same / different to being good at your multiplication facts?

How would you solve this?

A family has **£96.00** to spend at the adventure park. Each ride costs **£4.00**.
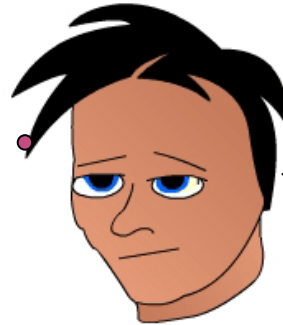
How many rides can the family go on?

Can you solve it in another way?

Discuss each strategy. Are they both thinking multiplicatively?

They think …

They record …

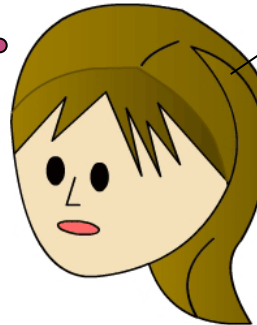I need to find out how many times I can take £4.00 away from £96.00

Frank

```
  96
 -4
 92
 -4
 88
 -4
 84
etc.  …
```

There are 25 lots of £4.00 in £100, £96.00 is £4.00 less, so…

Beth

25 x 4 = 100
100 – 4 = 96
So
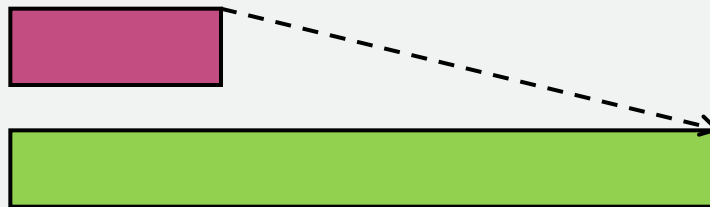24 x 4 = …

Multiplicative reasoning
– Key ideas

Two types of numerical relationship:

Additive

Multiplicative

Comparing additive and multiplicative relationships

Additive relationship

Multiplicative relationship

+8

×5

| 2 | 10 |

| 2 | 10 |

-8

÷5

Unitising

Fosnott and Dolk:

In order to reason multiplicatively, children need to be able to 'unitise' (treat a group as a single entity).

Each coin represents 2 – a value of 2 is attributed to each.

Building a sense of a unit

Real life units 1 bike
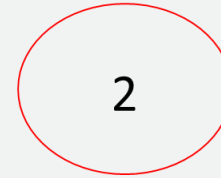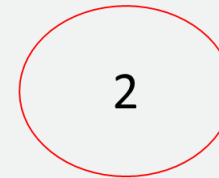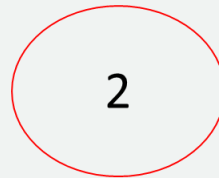
with two wheels

(packets or bags are useful for this)

Discrete objects collected

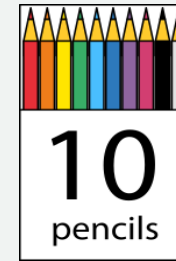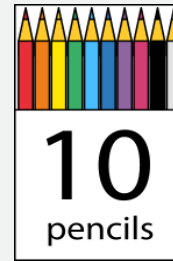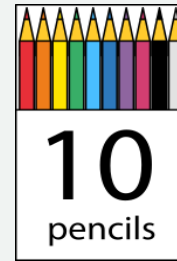into a given set

Objects in equal piles

Small steps in representation

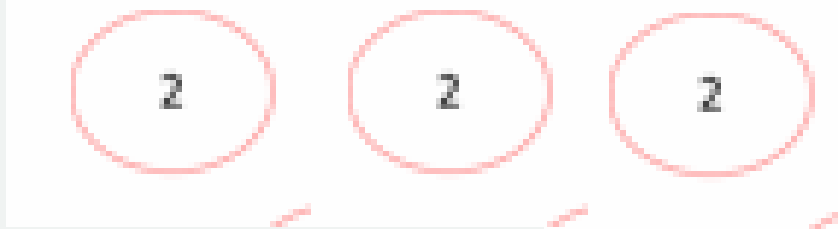Here are some representations that could be used in multiplication. What order would you introduce them in and why?

Small Steps in Representation

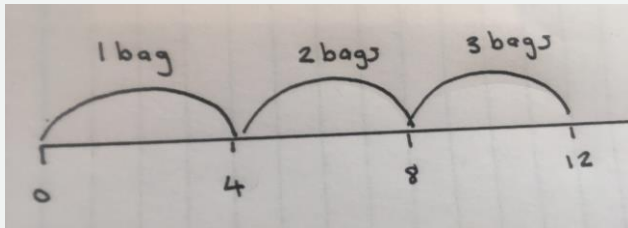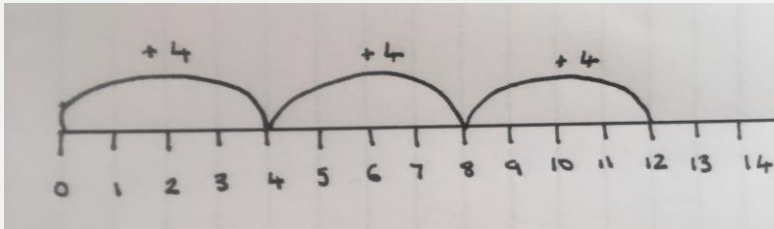Creating groups / Pictures separated physically

numbers separated physically

numbers in a bar model (abstract)

| 2 | 2 | 2 |

Multiplication as repeated addition

There are 4 apples in a bag. I buy 3 bags.
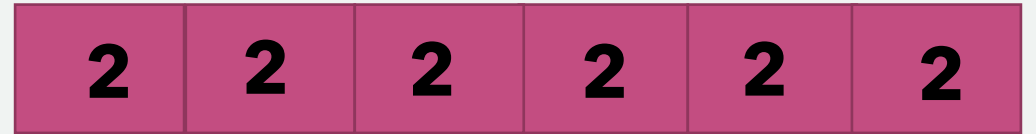*Altogether I buy 12 apples.*

I bag is 4, 2 bags are 8, 3 bags are 12

4, 8, 12

Moving from repeated addition to multiplication

Describe the number of people in the cars

2	2	2	2	2	2

2+2+2+2+2+2

There are 6 groups of 2

or 6 times 2   **6 x 2**

6 is the number of groups (cars) and 2 is the number of people in each group

3 times bigger is not the same as 3 lots of...
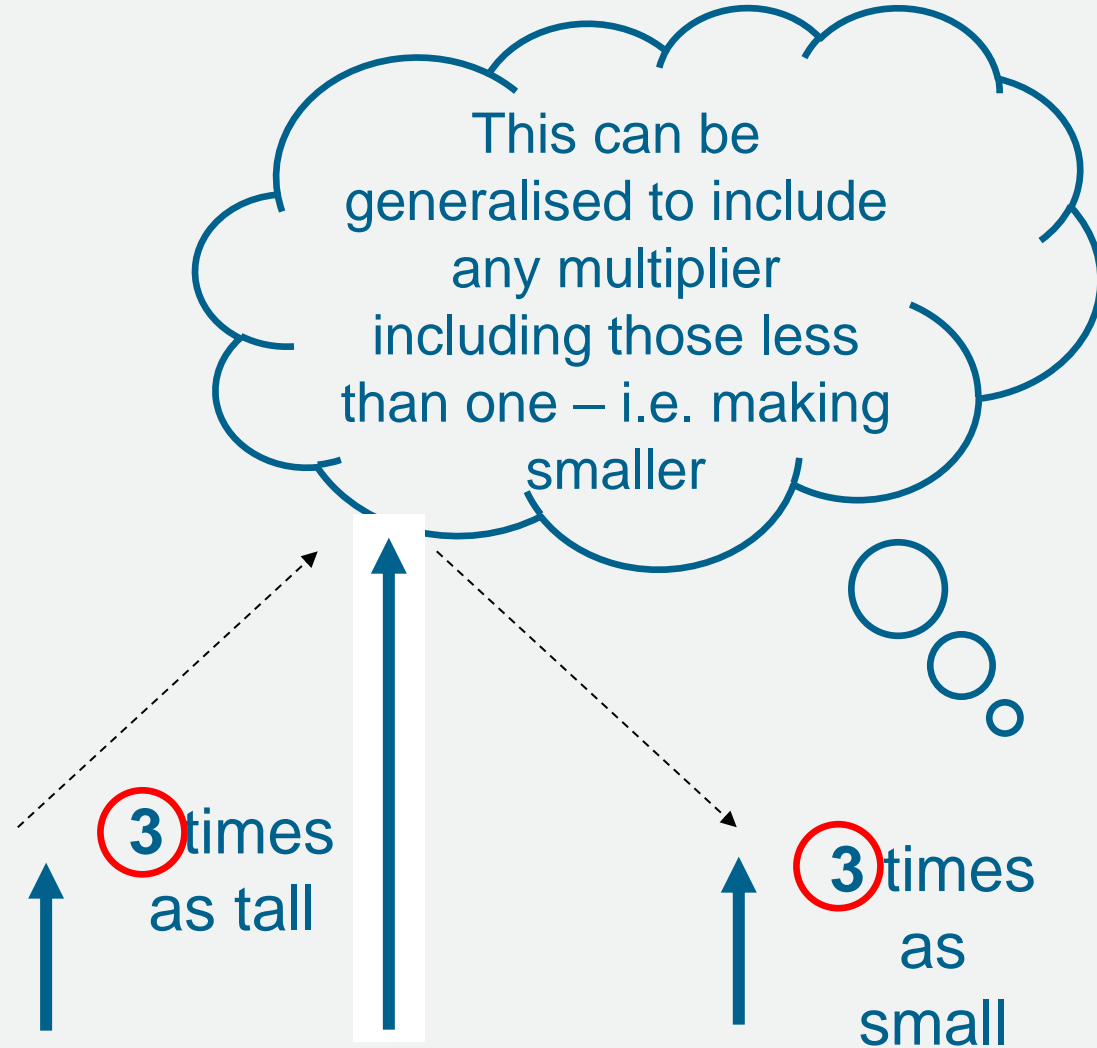

3 times as tall

Not 3 times as many

Scaling Models for multiplication

Scaling

3 times larger

3 times greater than

3 times more

This can be generalised to include any multiplier including those less than one – i.e. making smaller

3 times as tall

3 times as small

Multiplication as scaling: Measures

x times as far   x times as heavy   x times as much

Year 3 National Curriculum objective (multiplication and division):

solve problems, including missing number problems, involving multiplication and division, <mark>including positive integer scaling problems</mark>

With your partner, practice writing some 'positive integer scaling problems'
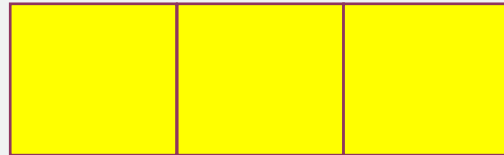
Scaling linked to Bar Model

There is a polar bear that is three times the weight of a baby elephant. Altogether they weigh 800kg. What is the weight of the polar bear?
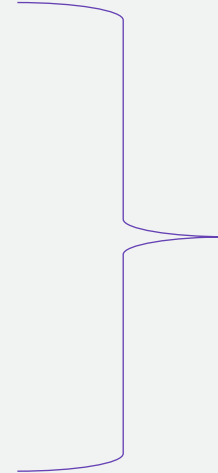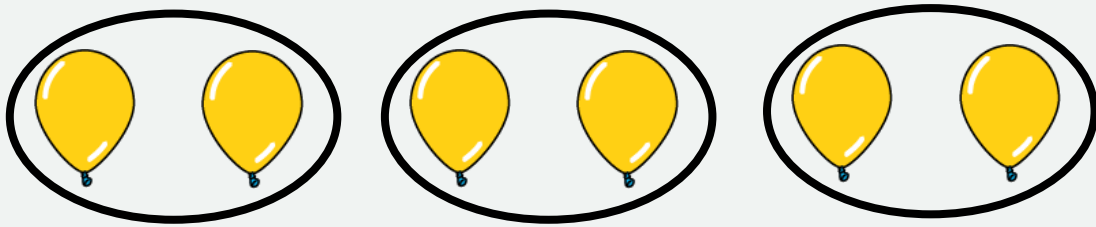
Baby elephant

Polar Bear

800kg

Structures of division

Quotitive division (grouping)

6 balloons
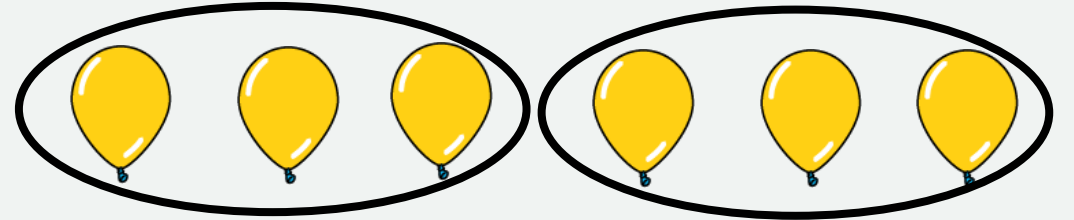**divided into groups of** 2.

$$6 \div 2 = 3$$

dividend

divisor

quotient

Partitive division (sharing)

6 balloons
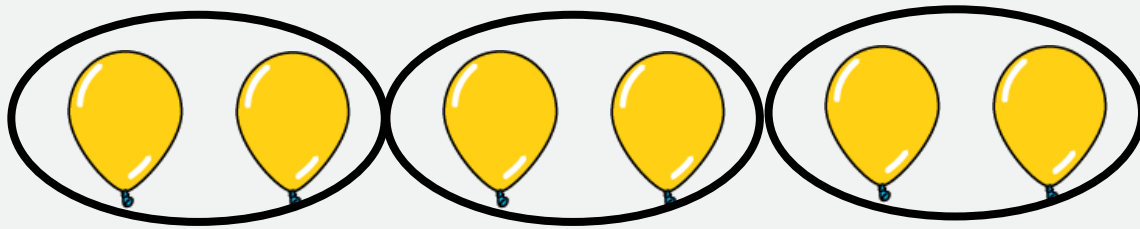**divided between** 2.

$$6 \div 2 = 3$$

dividend

divisor

quotient

Structures of division

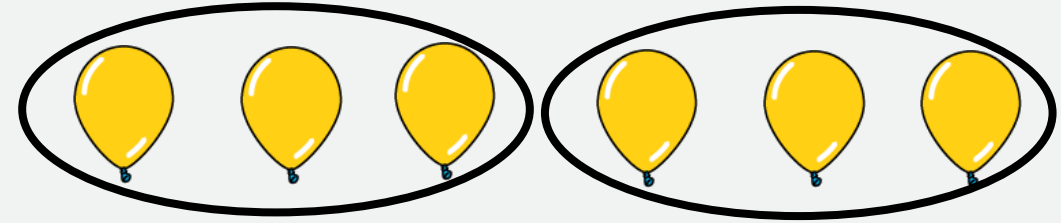6 balloons

divided into groups of 2.

$$6 \div 2 = 3$$

total quantity

group size
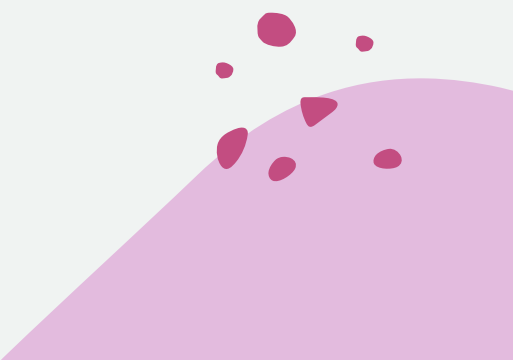
number of groups

6 balloons

divided between 2.

$$6 \div 2 = 3$$

total quantity
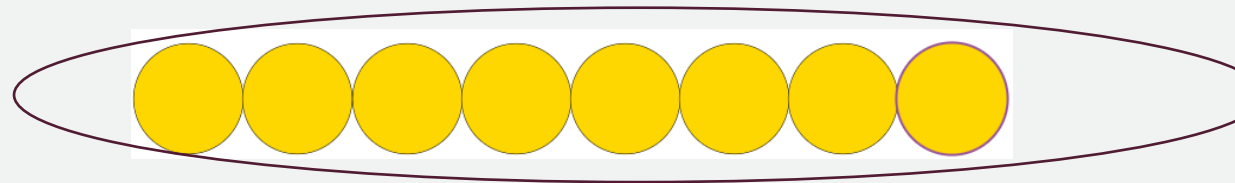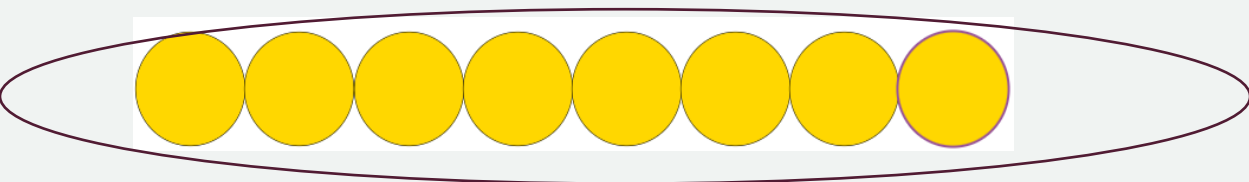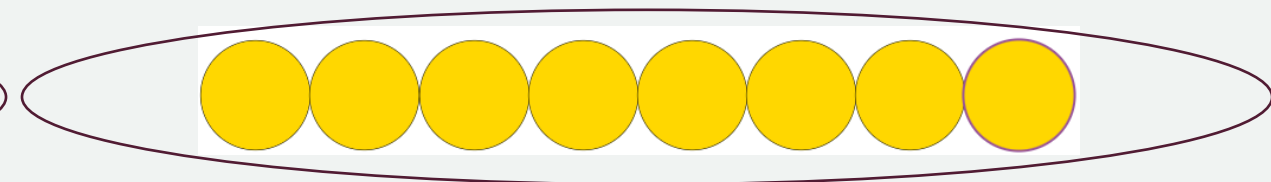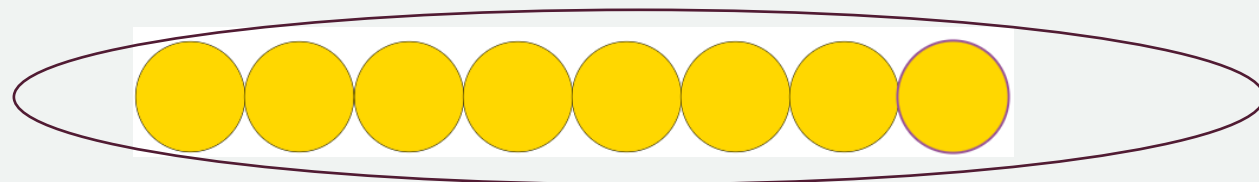
number of groups

group size

I need to buy 32 metres of fencing to go around my garden. The fencing is sold in 8- metre lengths. How many 8-metre lengths do I need to buy?

32m			
8	8	8	8

$$32 \div 8 = 4$$

dividend   divisor   quotient

Structures of division

$$32 \div 8 = 4$$

total
quantity

group
size

number of
groups

Quotitive (grouping) or Partitive (Sharing)

There are 24 strawberries in a tub. I share them equally between the 4 people in my family. How many does each person get?
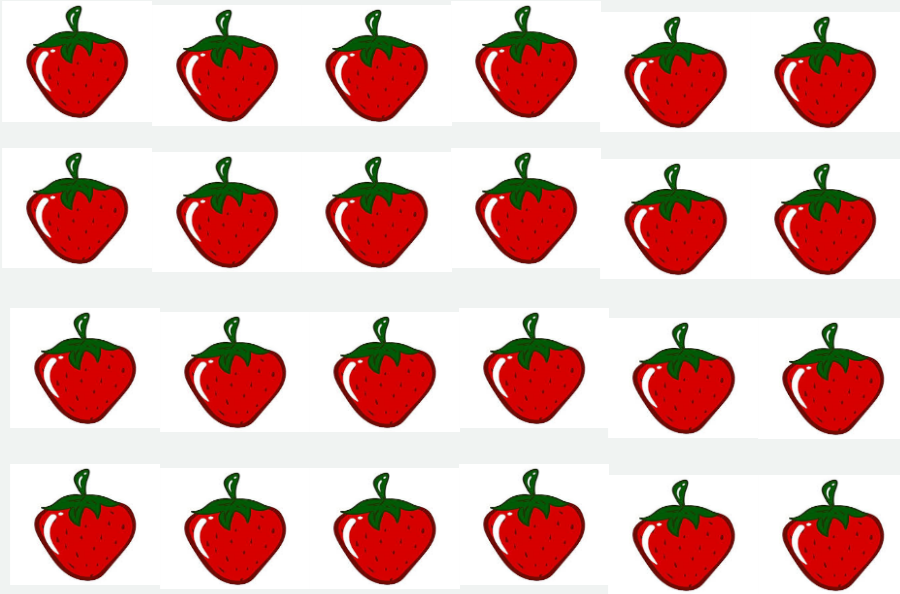
$$24 \div 4 = 6$$

Total Quantity

Number of groups

Group size

There are 8 children coming to the party (including Jola). They are all going to the cinema.

How many cars will they need to take them there? Each car can take 4 children and a driver.

Jola is going to give everyone some chocolate eggs to take home at the end of the party. They fit into egg boxes which hold 6 eggs each. Will 50 eggs be enough for each of the 8 party guests to have a box of eggs?

Arrays

Arrays are an incredibly powerful image for multiplicative thinking. They:

allow for exploration of the concept
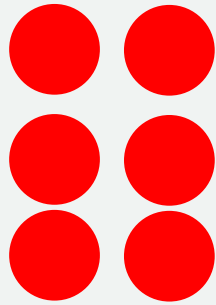
link multiplication to division

illustrate commutativity

illustrate the distributive law

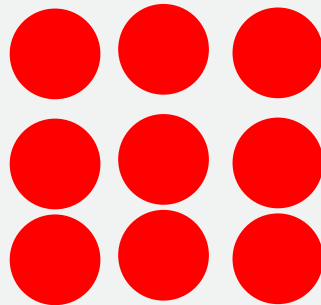Children can start to use arrays very quickly, once they have mastered unitising.

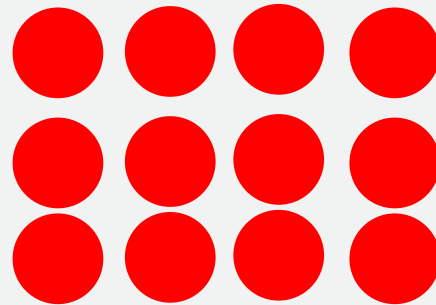# *Visualising building multiplication facts using arrays*

1 x 3
3 x 1

2 x 3
3 x 3

3 x 3
3 x 3

4 x 3
3 x 4

Factor x factor = product
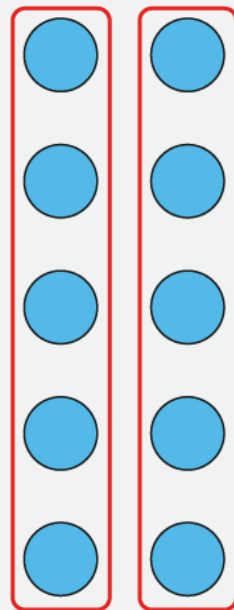
Commutativity is the idea that the numbers in a multiplication equation can be "swapped" and the total will be the same, e.g. 2 x 5 = 5 x 2

Commutativity

5 x 2 = 10
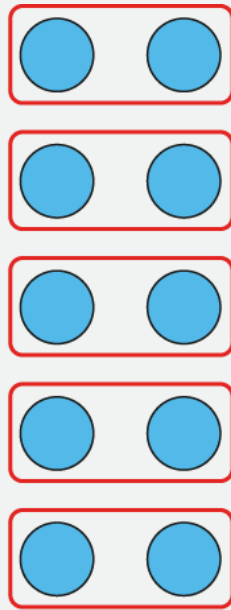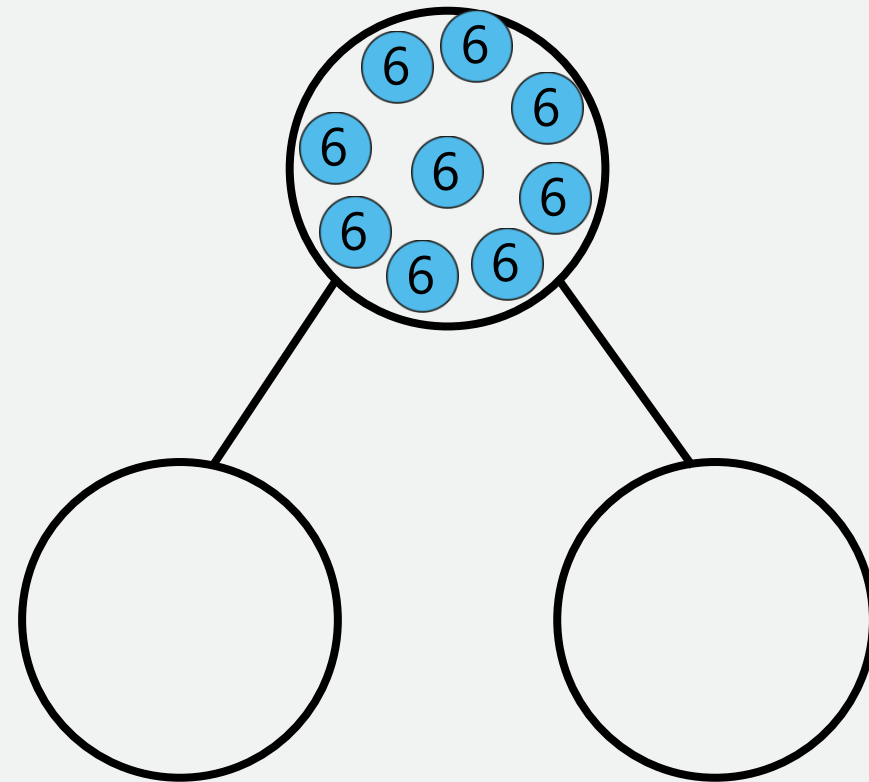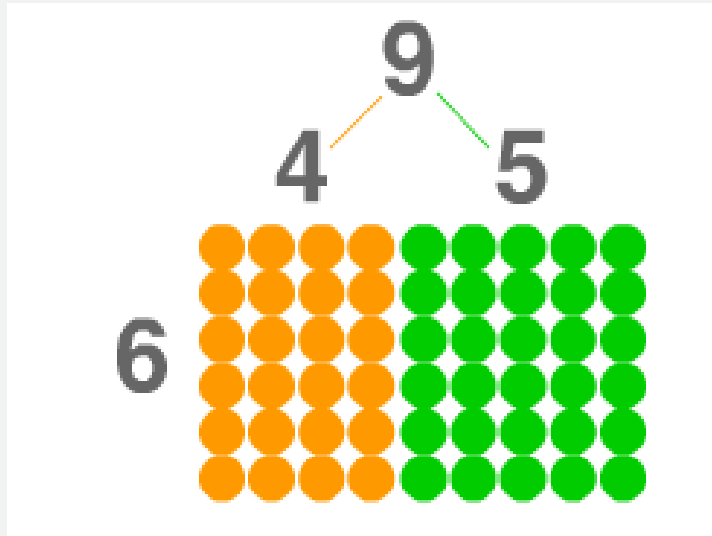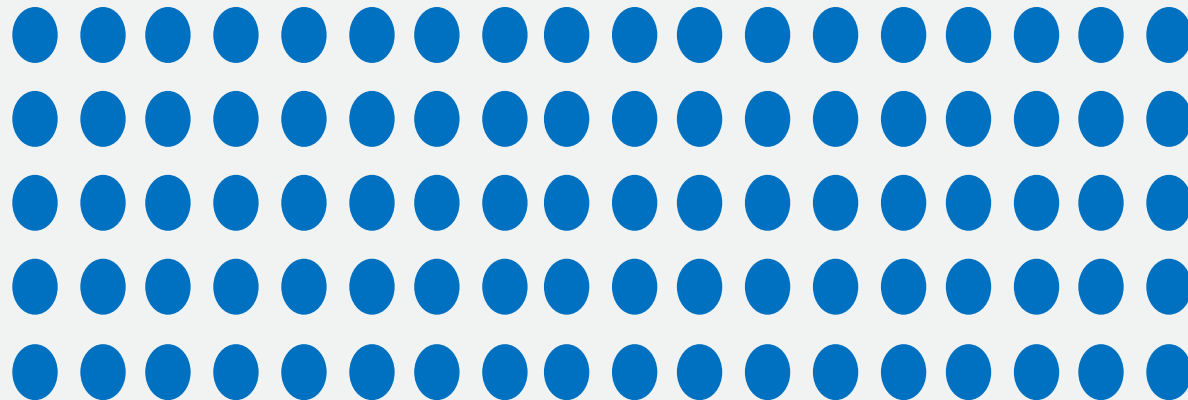

2 x 5 = 10

The distributive law: 6 x 9

Using relationships for fluent calculation

How would you solve 18 x 5?
Can you think about how it can be represented on this array

Key Take Aways

Joe Burbank Maths

X formally known as Twitter:
@joebur2009

E-mail:
joeburbankmathsconsulting@gmail.com

Website: www.joeburbankmaths.com